

# Weird Chemist

## DPP-7 : Average Atomic Mass

Chapter: Some Basic Concepts of Chemistry

Complete Solution Sheet — Q.1 to Q.5

### Core Formula: Weighted Average Atomic Mass

$$\bar{A} = \frac{m_1 \times a_1 + m_2 \times a_2 + m_3 \times a_3 + \dots}{100}$$

where  $m_i$  = mass number of isotope  $i$ ,  $a_i$  = % abundance of isotope  $i$ .

**Two-isotope shortcut** (when only two isotopes exist):

Let abundance of isotope 1 =  $x\%$ , then abundance of isotope 2 =  $(100 - x)\%$ .

$$\bar{A} = \frac{m_1 \cdot x + m_2 \cdot (100 - x)}{100}$$

Solve for  $x$ . This is the standard “reverse” technique for Q.4 and Q.5.

**Key concept:** The average atomic mass is a **weighted** average — heavier isotopes and more abundant isotopes both pull the average towards themselves. The average must always lie **between** the masses of the lightest and heaviest isotopes.

## TYPE : Average Atomic Mass

**Q.1 B has two isotopes  $^{10}\text{B}$  (19%) and  $^{11}\text{B}$  (81%). The atomic mass of B is**

### Explanation

Using the weighted average formula:

$$\bar{A} = \frac{(10 \times 19) + (11 \times 81)}{100} = \frac{190 + 891}{100} = \frac{1081}{100} = \mathbf{10.81}$$

### Approach / Analogy

Think of it like a weighted average of marks: if 19% of students score 10 and 81% score 11, the class average is pulled much closer to 11 (since more students scored it). Here,  $^{11}\text{B}$  is far more abundant (81%) so the average (10.81) is very close to 11. Real boron: atomic mass = 10.81 u — this is the actual value used in periodic tables!

### Common Mistake

Taking a simple average:  $(10 + 11)/2 = 10.5$ . This ignores the abundances and treats both isotopes as equally common. Abundances are percentages (or fractions) that act as weights — always multiply mass by its % before summing, then divide by 100.

### Answer

(1) 10.81

**Q.2** An element X has the following isotopic composition;  $^{200}\text{X}$ : 90%;  $^{199}\text{X}$ : 8.0%;  $^{202}\text{X}$ : 2.0%. The weighted average atomic mass of the naturally occurring element X is closest to: [AIPMT (Prelims)-2007]

### Explanation

$$\begin{aligned}\bar{A} &= \frac{(200 \times 90) + (199 \times 8) + (202 \times 2)}{100} \\ &= \frac{18000 + 1592 + 404}{100} = \frac{19996}{100} = 199.96 \approx \mathbf{200}\end{aligned}$$

### Approach / Analogy

Three isotopes, but  $^{200}\text{X}$  dominates at 90%. The average must be dominated by 200. Quick mental estimate: start from 200 and check the pull from  $^{199}\text{X}$  (pulls down by  $8 \times 1 = 8$  units distributed over 100) and  $^{202}\text{X}$  (pulls up by  $2 \times 2 = 4$  units over 100). Net shift from 200:  $(-8 + 4)/100 = -0.04$ . So  $\bar{A} \approx 200 - 0.04 = 199.96 \approx 200$ .

### Common Mistake

Adding all three mass numbers and dividing by 3:  $(200 + 199 + 202)/3 = 200.33$ . This gives the *simple* average (treating each isotope as equally abundant), not the *weighted* average. With 90% abundance at 200, the answer must be very close to 200 — the weighted approach confirms 199.96.

### Answer

(2) 200 amu

**Q.3** An element X has the following isotopic composition  $^{56}\text{X}$ : 90%,  $^{57}\text{X}$ : 8%,  $^{59}\text{X}$ : 2.0%. The weighted average atomic mass of the naturally occurring element X is closest to

### Explanation

$$\begin{aligned}\bar{A} &= \frac{(56 \times 90) + (57 \times 8) + (59 \times 2)}{100} \\ &= \frac{5040 + 456 + 118}{100} = \frac{5614}{100} = \mathbf{56.14}\end{aligned}$$

### Approach / Analogy

Same structure as Q.2 but with smaller mass numbers.  $^{56}\text{X}$  dominates at 90%, so the answer must be just above 56. Calculate:  $56 + (0 \times 90 + 1 \times 8 + 3 \times 2)/100 = 56 + (8 + 6)/100 = 56 + 0.14 = 56.14$ . This element X is Iron (Fe) —  $^{56}\text{Fe}$  is the most abundant isotope of iron!

### Common Mistake

Adding the mass numbers:  $56 + 57 + 59 = 172$  then dividing by 3 to get 57.33. This is the unweighted average. The correct weighted average (56.14) is much closer to 56 because 90% of atoms have mass 56.

### Answer

(1) 56.14 amu

**Q.4 Nitrogen occurs in nature in the form of two isotopes with atomic mass 14 and 15 respectively. If average atomic mass of nitrogen is 14.0067, the % abundance of  $^{14}\text{N}$  isotope is**

### Explanation

Let % abundance of  $^{14}\text{N} = x$ , then % abundance of  $^{15}\text{N} = (100 - x)$ .

Setting up the weighted average equation:

$$\frac{14x + 15(100 - x)}{100} = 14.0067$$

$$14x + 1500 - 15x = 1400.67$$

$$-x = 1400.67 - 1500 = -99.33$$

$$x = \mathbf{99.33\%}$$

### Approach / Analogy

“Reverse” problem: given the average, find the abundance. Set up the formula with  $x$  as the unknown abundance of  $^{14}\text{N}$ . The answer 99.33% makes perfect physical sense —  $^{14}\text{N}$  is overwhelmingly dominant (almost all nitrogen in nature is  $^{14}\text{N}$ ). The average 14.0067 is barely above 14, confirming that  $^{15}\text{N}$  is extremely rare ( $\approx 0.67\%$ ).

### Common Mistake

Setting up the equation with  $x$  as the fraction (0 to 1) instead of percentage (0 to 100) and not multiplying 14.0067 by 100 on the right side. Always be consistent: if abundances are in %, keep everything in % throughout. Use:  $\frac{14x + 15(100 - x)}{100} = 14.0067$ , where  $x$  is % (between 0 and 100), not a fraction (between 0 and 1).

### Answer

(1) 99.33%

**Q.5 Copper has two naturally occurring isotopes  $^{63}\text{Cu}$  and  $^{65}\text{Cu}$ . If the average atomic mass of copper is 63.546 u, then natural abundance of lighter isotope of Cu will be [NCERT Pg. 17]**

## Explanation

Lighter isotope =  $^{63}\text{Cu}$ . Let its abundance =  $x\%$ .  
Then abundance of  $^{65}\text{Cu}$  =  $(100 - x)\%$ .

$$\frac{63x + 65(100 - x)}{100} = 63.546$$
$$63x + 6500 - 65x = 6354.6$$
$$-2x = 6354.6 - 6500 = -145.4$$
$$x = \frac{145.4}{2} = 72.7\%$$

## Approach / Analogy

Reverse approach again. The average (63.546) is much closer to 63 than to 65, so  $^{63}\text{Cu}$  (the lighter isotope) must be far more abundant — 72.7% confirms this. The “lever rule” intuition: average lies at 63.546, which is  $(65 - 63.546)/(65 - 63) = 1.454/2 = 72.7\%$  of the way from 65 back to 63. Same answer!

### Lever rule shortcut:

$$\% \text{ of lighter isotope} = \frac{m_{\text{heavy}} - \bar{A}}{m_{\text{heavy}} - m_{\text{light}}} \times 100 = \frac{65 - 63.546}{65 - 63} \times 100 = \frac{1.454}{2} \times 100 = 72.7\%$$

## Common Mistake

Writing  $63x + 65(100 - x) = 63.546$  (forgetting to divide by 100). The weighted average formula divides the sum by 100 when percentages are used. Without dividing:  $63x + 6500 - 65x = 63.546$  gives  $-2x = -6436.454 \rightarrow x = 3218\%$  — clearly nonsensical. Always divide by 100 when using % abundances.

## Answer

(4) 72.7%

## — End of DPP-7 Complete Solution Sheet —

Average Atomic Mass · Q.1 to Q.5 · All Questions Complete

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