

Thermodynamics DPP-5 Solutions

Topic: Isothermal Process

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Q1. The work done by a weightless piston in causing expansion V (at constant T) when P is variable.

Conceptual Approach

Isothermal expansion mein jab pressure continuously change hota hai, toh work ko integration se nikalte hain: $W = -\int P dV$.

Final Answer: $W = -\int P dV$. (Option A)

Q2. The work done by 100 cal of heat in isothermal expansion of ideal gas.

Conceptual Approach

Isothermal mein $\Delta U = 0$. So $q = w$. Heat absorbed = Work done. Convert calories \rightarrow joules.

Stepwise Solution Step 1: Given $q = 100$ cal.

Step 2: 1 cal = 4.184 J. So 100 cal = 418.4 J.

Final Answer: Work done = 418.4 J. (Option A)

Q3. Work done in reversible isothermal expansion.

Conceptual Approach

Formula hai: $w = -2.303 nRT \log\left(\frac{V_f}{V_i}\right)$. Negative sign because system doing work.

Final Answer: $w = -2.303 nRT \log\left(\frac{V_f}{V_i}\right)$. (Option A)

Q4. x mol ideal gas at 27°C expands isothermally reversibly from V to $10V$. Find work done.

Conceptual Approach

Isothermal reversible work: $w = -nRT \ln\left(\frac{V_f}{V_i}\right)$.

Stepwise Solution Step 1: $n = x$, $T = 300$ K.

Step 2: $V_f/V_i = 10$.

Step 3: $w = -x(8.314)(300) \ln 10$ J.

Final Answer: $w = -300xR \ln 10$. (Option C)

Q5. Work done during isothermal reversible expansion of 5 mol ideal gas when volume doubles at 400 K.

Conceptual Approach

Formula: $w = -nRT \ln\left(\frac{V_f}{V_i}\right)$. If volume doubles $\rightarrow V_f/V_i = 2$.

Stepwise Solution Step 1: $n = 5$, $R = 8.314 \text{ J/mol-K}$, $T = 400 \text{ K}$.

Step 2: $w = -5 \times 8.314 \times 400 \times \ln 2$.

Step 3: $\ln 2 \approx 0.693$. So $w = -11520 \text{ J} = -11.5 \text{ kJ}$.

Final Answer: -11.5 kJ . (Option A)

Q6. Work done in ergs for a reversible expansion of 1 mol ideal gas from 10 L to 20 L at 25°C.

Conceptual Approach

Reversible isothermal work: $w = -2.303 nRT \log\left(\frac{V_f}{V_i}\right)$. Here units \rightarrow ergs, so $R = 8.31 \times 10^7 \text{ erg K}^{-1} \text{ mol}^{-1}$.

Stepwise Solution Step 1: $n = 1$, $T = 298 \text{ K}$, $V_f/V_i = 20/10 = 2$.

Step 2: $w = -2.303 \times 8.31 \times 10^7 \times 298 \times \log 2$.

Final Answer: $w = -2.303 \times 8.31 \times 10^7 \times 298 \log 2 \text{ ergs}$. (Option A)

Q7. A piston with 0.04 mol gas expands reversibly from 50 mL to 375 mL at 37°C, absorbing 208 J heat. Find q and w.

Conceptual Approach

Isothermal expansion $\rightarrow \Delta U = 0$. So $q = -w$. If heat absorbed = +208 J, then work = -208 J.

Final Answer: $q = +208 \text{ J}$, $w = -208 \text{ J}$. (Option A)

Q8. An ideal gas expands isothermally from 50 L to 100 L against 2 atm external pressure. Find W, U and q.

Conceptual Approach

Work (against constant P): $w = -P_{ext} \Delta V$. Isothermal $\rightarrow U = 0$, so $q = -w$.

Stepwise Solution Step 1: $\Delta V = 100 - 50 = 50 \text{ L}$.

Step 2: $w = -2 \times 50 = -100 \text{ L}\cdot\text{atm}$.

Step 3: $U = 0$. So $q = +100 \text{ L}\cdot\text{atm}$.

Final Answer: $w = -100 \text{ L}\cdot\text{atm}$, $U=0$, $q = +100 \text{ L}\cdot\text{atm}$. (Option A)

Q9. A gas at 300 K expands from 0.1 L to 0.25 L against 2 bar constant pressure. Find work done. (1 L·bar = 100 J)

Conceptual Approach

Work: $w = -P_{ext} \Delta V$. Convert L·bar \rightarrow J.

Stepwise Solution Step 1: $\Delta V = 0.25 - 0.10 = 0.15 \text{ L}$.

Step 2: $w = -2 \times 0.15 = -0.30 \text{ L}\cdot\text{bar}$.

Step 3: Convert: $-0.30 \times 100 = -30$ J.

Final Answer: -30 J. (Option A)

Q10. 2 mol gas expands isothermally from 5 dm³ to 40 dm³ against 2 atm constant pressure at 298 K. Find work.

Conceptual Approach

External pressure constant $\rightarrow w = -P_{ext} \Delta V$. Convert L·atm \rightarrow J (1 L·atm = 101.32 J).

Stepwise Solution Step 1: $\Delta V = 40 - 5 = 35$ L.

Step 2: $w = -2 \times 35 = -70$ L·atm.

Step 3: In J: $-70 \times 101.32 \approx -7091$ J.

Final Answer: -7091 J. (Option A)

Q11. Calculate w for isothermal reversible expansion of 1 mol ideal gas from P=1.0 bar to P=0.1 bar at 273 K.

Conceptual Approach

Reversible isothermal expansion work: $w = -2.303 nRT \log\left(\frac{P_i}{P_f}\right)$ (since $P_i/P_f = V_f/V_i$).

Stepwise Solution Step 1: $n = 1$, $R = 8.314$ J/mol-K, $T = 273$ K.

Step 2: $\log(P_i/P_f) = \log(1/0.1) = \log(10) = 1$.

Step 3: $w = -2.303 \times 8.314 \times 273 \times 1 \approx -5227$ J.

Final Answer: -5227 J. (Option A)

Q12. At 27°C, 1 mol ideal gas compressed isothermally and reversibly from 2 atm to 10 atm. Find q in cal.

Conceptual Approach

Isothermal: $U = 0 \rightarrow q = -w$. Work: $w = -2.303 nRT \log\left(\frac{V_f}{V_i}\right) = -2.303 nRT \log\left(\frac{P_i}{P_f}\right)$.

Stepwise Solution Step 1: $T = 300$ K, $n = 1$.

Step 2: $\log(P_f/P_i) = \log(10/2) = \log 5 = 0.699$.

Step 3: $w = -2.303 \times 8.314 \times 300 \times 0.699 \approx -4042$ J.

Step 4: In cal: $-4042/4.184 \approx -966$ cal. So $q = +966$ cal (since $U=0$).

Final Answer: $+965.9$ cal. (Option B)

Q13. 2 L of N₂ at 0°C, 5 atm expands isothermally against 1 atm constant pressure until final P = 1 atm. Find work.

Conceptual Approach

Work against constant external P: $w = -P_{ext} \Delta V$. Use Boyle's law to find final volume.

Stepwise Solution Step 1: $P_1V_1 = P_2V_2$. $5 \times 2 = 1 \times V_2 \Rightarrow V_2 = 10$ L.

Step 2: $\Delta V = 10 - 2 = 8$ L.

Step 3: $w = -1 \times 8 = -8 \text{ L}\cdot\text{atm}$.

Step 4: In J: $-8 \times 101.32 = -810.6 \text{ J}$.

Final Answer: -810 J . (Option D)

Q14. 1 mol gas A expands from 2 L \rightarrow 4 L isothermally (reversible). 3 mol gas B expands from 2 L \rightarrow x L with same work. Find x.

Conceptual Approach

Work reversible isothermal: $w = -nRT \ln(V_f/V_i)$. Equating $w_A = w_B$.

Stepwise Solution Step 1: For A: $w_A = -1RT \ln(4/2) = -RT \ln 2$.

Step 2: For B: $w_B = -3RT \ln(x/2)$.

Step 3: Equating: $-RT \ln 2 = -3RT \ln(x/2)$.

Step 4: $\ln(x/2) = \frac{1}{3} \ln 2 \Rightarrow x/2 = 2^{1/3}$.

Step 5: $x = 2 \times 2^{1/3}$