

Heisenberg's Uncertainty Principle (solution)

1. The uncertainty in position of electron and helium atom are same. If uncertainty in momentum for the electron is 3.2×10^6 (same units), then the uncertainty in momentum of the helium atom will be —

Explanation (simple words): Heisenberg ke hisaab se $\Delta x \Delta p \geq \frac{h}{4\pi}$. Jab Δx same ho, to minimum Δp same rehta hai, mass par depend nahi karta.

Approach: Equal $\Delta x \Rightarrow$ equal minimum Δp .

Answer: $\boxed{3.2 \times 10^6}$ (same units as given).

2. Calculate uncertainty in position of an electron (mass 9.1×10^{-28} g) moving at 3.0×10^4 cm s⁻¹, if the uncertainty in velocity is 0.011%. ($h = 6.626 \times 10^{-27}$ erg s)

Explanation (simple words): Pehle Δv nikalo, phir $\Delta x = \frac{h}{4\pi m \Delta v}$ (cgs units).

Approach: $\Delta v = 0.011\% \times 3.0 \times 10^4 = 3.3$ cm s⁻¹; $m = 9.1 \times 10^{-28}$ g.

Steps:

$$\Delta x = \frac{6.626 \times 10^{-27}}{4\pi (9.1 \times 10^{-28}) (3.3)} = \boxed{1.76 \times 10^{-1} \text{ cm}} \approx \boxed{0.175 \text{ cm}}$$

Why this formula? $\Delta x \Delta p = \frac{h}{4\pi}$ and $\Delta p = m \Delta v$.

3. Heisenberg's uncertainty principle is *not significant* for — (i) motor car, (ii) stationary particle

Explanation (simple words): Macroscopic bodies (motor car) ke liye effect itna chhota hota hai ki practically negligible hai. Completely stationary particle ki exact position-momentum ek saath define karna physically meaningful nahi.

Answer: $\boxed{(i) \& (ii) \text{ both}}$.

4. What should be the momentum of a particle if its de Broglie wavelength is 1 Å? ($h = 6.6252 \times 10^{-27}$ erg s)

Explanation (simple words): de Broglie: $\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda}$.

Steps:

$$\lambda = 1 \text{ \AA} = 10^{-8} \text{ cm}, \quad p = \frac{6.6252 \times 10^{-27}}{10^{-8}} = \boxed{6.6252 \times 10^{-19} \text{ g cm s}^{-1}}$$

Why this formula? Matter waves relation directly links momentum and wavelength. .

5. Mass of a sodium photon of wavelength 5894 Å ($c = 3 \times 10^8$ m s⁻¹, $h = 6.6252 \times 10^{-34}$ J s) is —

Explanation (simple words): Photon rest mass zero hota hai, but *relativistic mass equivalent*

$$m = E/c^2 = \frac{h}{\lambda c} \text{ nikal sakte hain.}$$

Steps:

$$\lambda = 5.894 \times 10^{-7} \text{ m, } m = \frac{6.6252 \times 10^{-34}}{(5.894 \times 10^{-7})(3 \times 10^8)} = \boxed{3.75 \times 10^{-36} \text{ kg}}$$

Why this formula? $E = hc/\lambda$ and $E = mc^2$.

6. Uncertainty in position of a 0.25 g particle is 10^{-5} m. Find the uncertainty in its velocity. ($h = 6.6 \times 10^{-34}$ J s)

Explanation (simple words): Macroscopic mass ke liye Δv bahut chhota aata hai.

Approach: $\Delta v = \frac{h}{4\pi m \Delta x}$ with $m = 2.5 \times 10^{-4}$ kg, $\Delta x = 10^{-5}$ m.

Steps:

$$\Delta v = \frac{6.6 \times 10^{-34}}{4\pi (2.5 \times 10^{-4})(10^{-5})} = \boxed{2.1 \times 10^{-26} \text{ m s}^{-1}}$$

Why this formula? $\Delta p = m\Delta v$ and $\Delta x \Delta p = \frac{h}{4\pi}$.

7. The position of both an electron and a helium atom is known within 1.0 nm. If the momentum of the electron is known within 5.0×10^{-26} kg m s⁻¹, what is the *minimum* uncertainty in the momentum of the helium atom?

Explanation (simple words): Minimum Δp sirf Δx par depend karta hai: $\Delta p_{\min} = \frac{h}{4\pi \Delta x}$.

Steps:

$$\Delta x = 1.0 \times 10^{-9} \text{ m, } \Delta p_{\min} = \frac{6.626 \times 10^{-34}}{4\pi \times 10^{-9}} = \boxed{5.27 \times 10^{-26} \text{ kg m s}^{-1}}$$

Why this formula? Heisenberg bound mass-se independent hota hai (given same Δx). .

8. If the uncertainties in position and momentum are (numerically) equal, then the uncertainty in velocity is —

Explanation (simple words): Diya hai $\Delta x = \Delta p$ (numerically). Also $\Delta x \Delta p = \frac{h}{4\pi}$ and $\Delta p = m \Delta v$.

Steps:

$$(\Delta x)^2 = \frac{h}{4\pi} \Rightarrow \Delta x = \sqrt{\frac{h}{4\pi}}$$

$$\Delta v = \frac{\Delta p}{m} = \frac{\Delta x}{m} = \boxed{\frac{1}{m} \sqrt{\frac{h}{4\pi}}} \text{ (formal result)}$$

Why this formula? Equal numerical uncertainties ke assumption se direct relation milta hai; dimensions practically mix units, isliye isko sirf algebraic condition ke roop me dekhein.

9. Measurement of electron position gives uncertainty in momentum $\Delta p = 1.0 \times 10^{-18}$ g cm s⁻¹. Find the uncertainty in its velocity. ($m_e = 9.0 \times 10^{-28}$ g)

Approach: $\Delta v = \frac{\Delta p}{m}$ in cgs.

Steps:

$$\Delta v = \frac{1.0 \times 10^{-18}}{9.0 \times 10^{-28}} = 1.11 \times 10^9 \text{ cm s}^{-1} \approx \boxed{1.0 \times 10^9 \text{ cm s}^{-1}}$$

Why this formula? Momentum per unit mass is velocity.

10. The uncertainty in momentum of an electron is 1.0×10^{-5} kg m s⁻¹. The uncertainty in its position is ($h = 6.62 \times 10^{-34}$ J s) —

Approach: $\Delta x = \frac{h}{4\pi \Delta p}$.

Steps:

$$\Delta x = \frac{6.62 \times 10^{-34}}{4\pi \times 10^{-5}} = \boxed{5.27 \times 10^{-30} \text{ m}}$$

Why this formula? Direct Heisenberg bound inversion.

11. The uncertainties in velocities of A and B are 0.05 and 0.02 m s⁻¹. If $m_B = 5m_A$, find $\frac{\Delta x_A}{\Delta x_B}$.

Approach: $\Delta x \propto \frac{1}{m \Delta v}$, so ratio = $\frac{m_B \Delta v_B}{m_A \Delta v_A}$.

Steps:

$$\frac{\Delta x_A}{\Delta x_B} = \frac{5m_A \times 0.02}{m_A \times 0.05} = \boxed{2}$$

Why this formula? Heisenberg bound equality case.

12. For an electron, if uncertainty in velocity is Δv , then uncertainty in position is —

Answer: $\Delta x = \frac{h}{4\pi m_e \Delta v}$.

Why this formula? $\Delta x \Delta p = \frac{h}{4\pi}$ and $\Delta p = m_e \Delta v$.

13. An atom has mass 0.02 kg and uncertainty in velocity 9.218×10^{-6} m s⁻¹. Find the uncertainty in position. ($h = 6.626 \times 10^{-34}$ J s) [AIEEE 2002]

Steps:

$$\Delta x = \frac{6.626 \times 10^{-34}}{4\pi (0.02)(9.218 \times 10^{-6})} = \boxed{2.86 \times 10^{-28} \text{ m}}$$

14. Uncertainty in position of a 25 g particle is 10^{-5} m. Hence the uncertainty in its velocity is — ($h = 6.6 \times 10^{-34}$ J s) [AIEEE 2002]

Steps:

$$\Delta v = \frac{6.6 \times 10^{-34}}{4\pi(0.025)(10^{-5})} = \boxed{2.1 \times 10^{-28} \text{ m s}^{-1}}$$

15. An electron ($m = 9.1 \times 10^{-31}$ kg) moves with $v = 300 \text{ m s}^{-1}$ accurate to 0.001%. Find uncertainty in position. ($h = 6.63 \times 10^{-34}$ J s)

Approach: $\Delta v = 0.001\% \times 300 = 3.0 \times 10^{-3} \text{ m s}^{-1}$.

Steps:

$$\Delta x = \frac{6.63 \times 10^{-34}}{4\pi(9.1 \times 10^{-31})(3.0 \times 10^{-3})} = \boxed{1.93 \times 10^{-2} \text{ m}}$$

$$x = \boxed{1.93 \times 10^{-3} \text{ m}}.$$

16. In an atom, an electron moves with speed 600 m s^{-1} with accuracy 0.005%. The certainty with which its position can be located is — ($h = 6.6 \times 10^{-34}$ J s)

Approach: $\Delta v = 0.005\% \times 600 = 3.0 \times 10^{-2} \text{ m s}^{-1}$.

Steps:

$$\Delta x = \frac{6.6 \times 10^{-34}}{4\pi(9.1 \times 10^{-31})(3.0 \times 10^{-2})} = \boxed{1.92 \times 10^{-3} \text{ m}}$$

$$x = \boxed{9.6 \times 10^{-3} \text{ m}}.$$

17. Uncertainty in position of an electron ($m = 9.1 \times 10^{-28}$ g) moving at $3.0 \times 10^4 \text{ cm s}^{-1}$ accurate up to 0.001% is — (Use $h/4\pi$ with $h = 6.626 \times 10^{-27}$ erg s)

Approach: cgs calculation: $\Delta v = 0.001\% \times 3.0 \times 10^4 = 0.3 \text{ cm s}^{-1}$.

Steps:

$$\Delta x = \frac{6.626 \times 10^{-27}}{4\pi(9.1 \times 10^{-28})(0.3)} = \boxed{1.93 \text{ cm}}$$

Why this formula? Same Heisenberg formula in cgs units.
